MATH 567: Mathematical Techniques in Data Science
Overview

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Admin

- Course website: `http://www.math.udel.edu/~dguillot/`
(Just google "Dominique Guillot")
- Slides will be posted in advance (will do my best).
- Textbook: *An Introduction to Statistical Learning*
by James, Witten, Hastie, and Tibshirani (50-75$).
Free pdf: `http://www-bcf.usc.edu/ gareth/ISL/`.
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- We will have a lab most Wednesdays (mandatory, some will count for credits). Bring your laptop. People who are auditing the class are expected to participate.
- Teamwork is encouraged. Keep in mind, however, that you will be alone during the exams (a large part of them will involve programming/numerical work).

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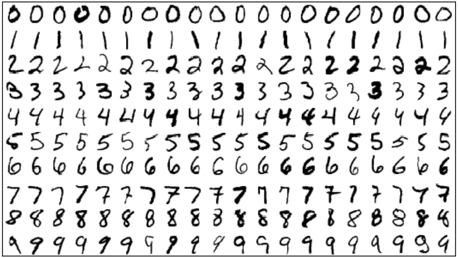
Note: output can be continuous (regression) or discrete (classification).

- Handwritten digits



- You are provided a dataset containing images (16 x 16 grayscale images say) of digits.
- Each image contains a single digit.
- Each image is labelled with the corresponding digit.
- Can think of each image as a vector in $X \in \mathbb{R}^{256}$ and the label as a scalar $Y \in \{0, \ldots, 9\}$.
- Idea: with a large enough sample, we should be able to *learn* to identify/predict digits.

Examples (cont.)

Gene expression data: rows = genes, columns = sample.

ESL, Figure 1.3.

- DNA microarrays measure the expression of a gene in a cell.
- Nucleotide sequences for a few thousand genes are printed on a glass slide.
- Each "spot" contains millions of identical molecules which will bind a specific DNA sequence.
- A target sample and a reference sample are labeled with red and green dyes, and each are hybridized with the DNA on the slide.
- Through fluoroscopy, the log (red/green) intensities of RNA hybridizing at each site is measured.

Question: do certain genes show very high (or low) expression for certain cancer samples?

- Information from 4601 email messages, in a study to screen email for "spam" (i.e., junk email).
- Data donated by George Forman from Hewlett-Packard laboratories.

TABLE 1.1. *Average percentage of words or characters in an email message equal to the indicated word or character. We have chosen the words and characters showing the largest difference between* `spam` *and* `email`.

	george	you	your	hp	free	hpl	!	our	re	edu	remove
spam	0.00	2.26	1.38	0.02	0.52	0.01	0.51	0.51	0.13	0.01	0.28
email	1.27	1.27	0.44	0.90	0.07	0.43	0.11	0.18	0.42	0.29	0.01

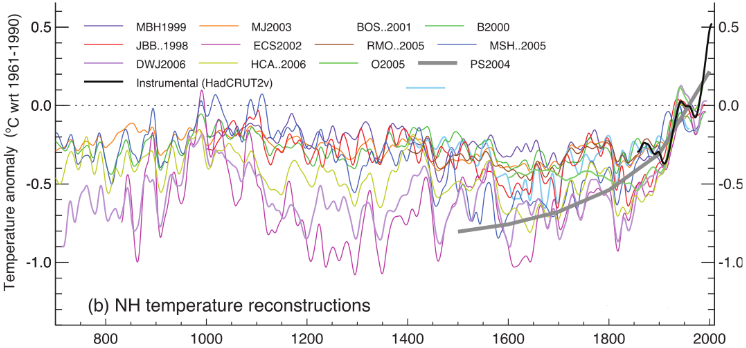
ESL, Table 1.1.

- Each message is labelled as spam/email.
- Want to predict the label using characteristics such as word counts.
- Which words or characters are good predictors?

Note: labelling data can be very tedious/expensive. Not always available.

Inferring the climate of the past:

- We have about 150 years of instrumental temperature data.
- Many things on Earth (proxies) record temperature indirectly (e.g. tree rings width, ice cores, sediments, corals, etc.).
- Want to infer the climate of the past from overlapping measurements.

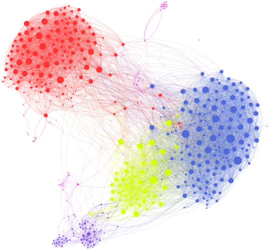


IPCC AR4.

Clustering:

$\rightarrow$

Wikipedia - Chire.



- Unsupervised problem.
- Work only with features/independent variables.
- Want to label points according to a measure of their similarity.

Modern problems

In modern problems:

- Dimension p is often very large.
- Sample size n is often very small compared to p.

In classical statistics:

- It is often assumed a lot of samples are available.
- Most results are asymptotic ($n \to \infty$).
- Generally not the right setup for modern problems.

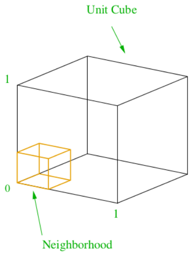
How do we deal with the $p \gg n$ case?

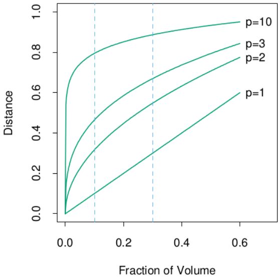
The curse of dimensionality

- Consider a hypercube with sides of length c along the axes in a unit hypercube. Its volume is c^p. To capture a fraction r of the unit hypercube:

$$c^p = r.$$

  Thus, $c = r^{1/p}$.
- A small sample of points in the hypercube will not cover a lot of the space.
- If $p = 10$, in order to capture 10% of the volume, we need $c \approx 0.8$!





The $p \gg n$ case: sparsity

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3. Climate reconstructions: large number of grid points, few annual observations. Can exploit conditional independence relations within the data.

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A linear regression problem: suppose we try to use linear regression to estimate Y (response) using X (predictors)

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- How do we identify the "right" subset of predictors?
- We can't examine all the $\binom{p}{k}$ possibilities! For example, $\binom{1000}{25} \approx 2.7 \times 10^{49}$!

Programming

We will use **R** to program, analyse data, etc. during the semester.

- Free. Open-source.
- Interpreted.
- A LOT of statistical packages.

If you have used Python or Matlab before:

`http://mathesaurus.sourceforge.net/octave-r.html`
`http://mathesaurus.sourceforge.net/r-numpy.html`

Homework

Get started with R:

- Download R at:

  `https://cran.r-project.org/`

- Download RStudio (free version):

  `https://www.rstudio.com/products/rstudio/download/`

- Install the packages `ISLR` and `MASS`. (Tools/Install packages... in RStudio)

- Tutorial:

  `http://tryr.codeschool.com`

Do (at least) the first 3 parts before the class on Wednesday.

Reading: Chapter 1 of the book.