MATH 829: Introduction to Data Mining and Analysis
Categorical data

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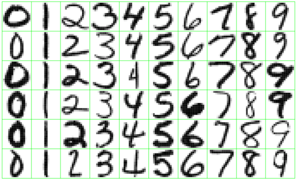

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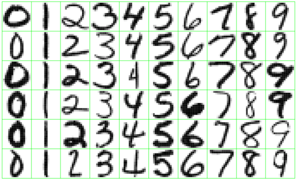

ESL, Figure 1.2.

- We begin with two very simple approaches: linear regression and nearest neighbors.

Linear regression

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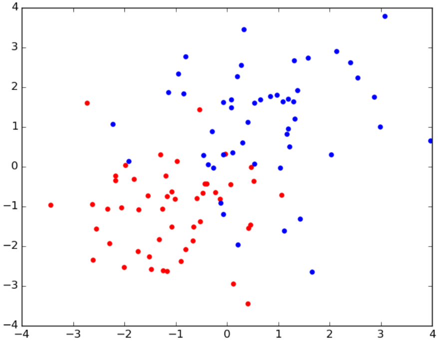
(0.1,0.4)	1
(0.5,0.8)	0
(0.6, 0.2)	1
$\vdots$	$\vdots$



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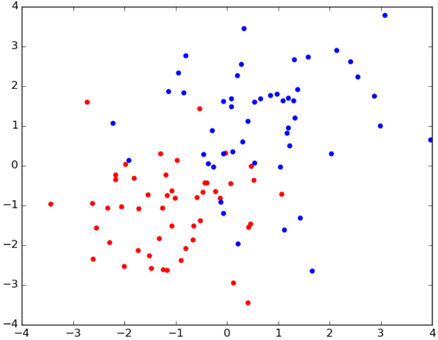

We want to predict new points' category.

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- Use

$$\hat{y} = \begin{cases} 0 & \text{if } x^T\beta < 0.5 \\ 1 & \text{if } x^T\beta \geq 0.5 \end{cases}.$$

Linear regression (cont.)

```
# X = 2*n by 2, Y = 2*n by 1 {0,1} labels
# Include an intercept
Xp = np.ones((2*n,3))
Xp[:,1:3] = X

# Use regression
beta = np.linalg.lstsq(Xp,Y)[0]

# Our decision boundary is
# beta[0] + beta[1] *x + beta[2]*y = 0.5,
# or y = (0.5-beta[0]-beta[1]*x)/beta[2]
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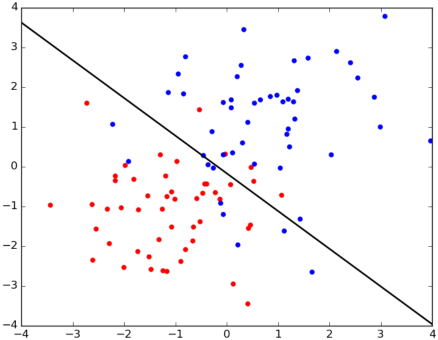
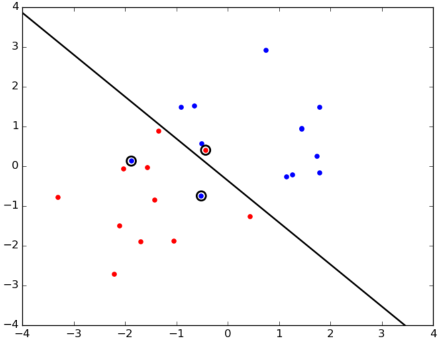


Test error

- As usual, we split our data into train and test sets.
- Train model on training set.
- Compute classification error on test set.

```
Xp_train, Xp_test, y_train, y_test =
  train_test_split(Xp, Y, test_size=0.25)
beta = np.linalg.lstsq(Xp_train,y_train)[0]
Y_hat = Xp_test.dot(beta)
```



Test set predicted labels

In general, when working with k categories, can use a loss-function

$$(L(i,j))_{i,j=1}^{k},$$

where $L(i,j)$ = cost for classifying i as j.

Nearest neighbors

Nearest neighbors: use closest observations in the training set to make predictions.

$$\hat{Y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i.$$

Here $N_k(x)$ denotes the k-nearest neighbors of x (w.r.t. some metric, e.g. Euclidean distance).

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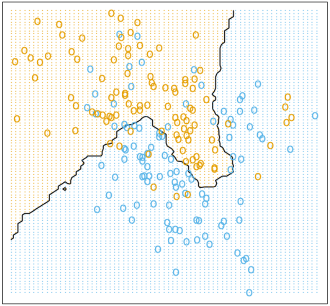
Here $N_k(x)$ denotes the k-nearest neighbors of x (w.r.t. some metric, e.g. Euclidean distance). Use a "majority vote" to determine final labels

$$\hat{G}(x) = \begin{cases} 0 & \text{if } \hat{Y}(x) < 0.5 \\ 1 & \text{if } \hat{Y}(x) \geq 0.5 \end{cases}.$$

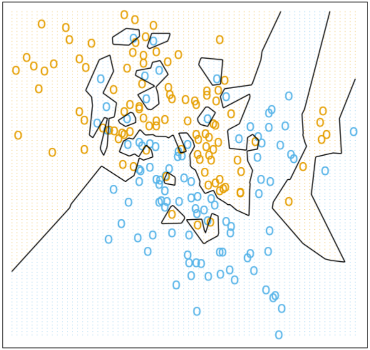


ESL, Fig. 2.2: 15 Nearest Neighbor classifier

Nearest neighbors

Reducing the number of neighbors leads to:

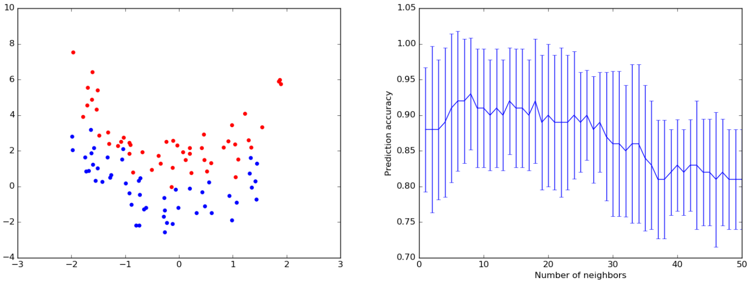
- A smaller training error (training error is 0 when using $k = 1$ neighbor).
- Can use train/test sets to choose k.
- Although a small k leads to a small training error, the model may not generalize well (large test error).



ESL, Fig. 2.3, 1 Nearest classifier

Example

```
from sklearn.neighbors import KNeighborsClassifier
neigh = KNeighborsClassifier(n_neighbors=i)
```




```
for i in range(1,N_max+1):
    neigh = KNeighborsClassifier(n_neighbors=i)
    for j, (train, test) in enumerate(kf):
        X_train, X_test, y_train, y_test =  X[train], X[test],
                                            Y[train], Y[test]
        neigh.fit(X_train,y_train)
        scores[i-1,j] = neigh.score(X_test, y_test)
```

Linear regression vs Nearest neighbors

A bias-variance tradeoff:

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- Adaptive, less assumptions on the data.
- A particular decision may depend only on a handful of points. Less robust.
- More wiggly and unstable.
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Many strategies exist to improve these methods (as we will see later).